

Year 1 Pure Chapter 8 - Binomial Expansion

Binomial Expansion

$$(a+b)^0 = a^0 + b^0$$

$$(a+b)^1 = a^1 + b^1$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots$$

.....

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

Identifying Values

$$(1 + 2x)^{10} \quad a=1 \quad b=2x \quad n=10$$

$$(10 - \frac{1}{2}x)^6 \quad a=10 \quad b=-\frac{1}{2}x \quad n=6$$

$$(x - \frac{1}{x})^5 \quad a=x \quad b=-\frac{1}{x} \quad n=5$$

Comparing Coefficients

$$1 + \underbrace{8q}_\substack{\uparrow \\ \text{coefficient} \\ \text{of } x} x + \underbrace{28q^2}_\substack{\uparrow \\ \text{coefficient} \\ \text{of } x^2} x^2$$

x^2 coefficient is double the coefficient of x

$$2(8q) = 28q^2$$

$$16q = 28q^2$$

$$16 = 28q$$

$$16$$

$$q = \frac{16}{28}$$

Binomial Expansion Estimation

Estimate 0.975^{10} to 4dp

Given

$$\left(1 - \frac{x}{4}\right)^{10} = 1 - 2.5x + 2.8125x^2 - 1.875x^3$$

$$0.975 = 1 - \frac{x}{4}$$

$$x = 0.1$$

Now sub $x=0.1$ into the binomial expansion.