1

$$x^2 + 6x + 18 > 2 - \frac{1}{2}x$$

$$x^2 + 6.5x + 16 > 0$$

complete the square

$\frac{1}{2}$

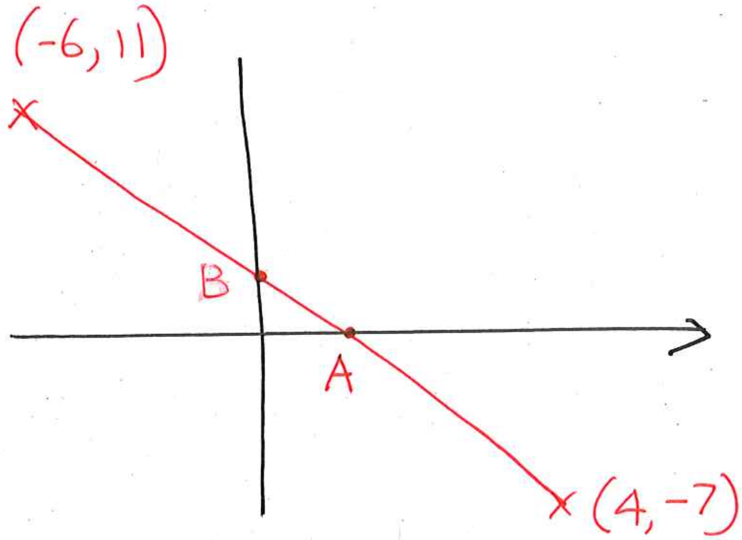
$$(x + 3.25)^2 - 3.25^2 + 16 > 0$$

$$(x + 3.25)^2 + 5.4375 > 0$$

any number squared is positive (or zero)

hence $(x+3.25)^2 + 5.4375 > 0$

(2)



a)

$$\text{gradient} = \frac{-18}{10} = \frac{-9}{5}$$

$$y = -\frac{9}{5}x + c$$

$$11 = -\frac{9}{5}(-6) + c$$

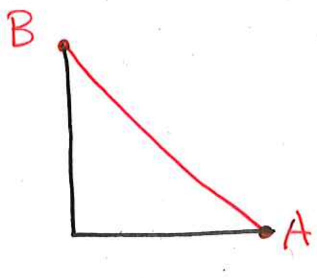
$$c = \frac{1}{5}$$

$$y = -\frac{9}{5}x + \frac{1}{5}$$

$$5y = -9x + 1$$

$$5y + 9x - 1 = 0$$

b)



A $y=0$ $\quad 5(0) + 9x - 1 = 0 \quad x = \frac{1}{9}$

B $x=0$ $\quad 5y + 0 - 1 = 0 \quad y = \frac{1}{5}$

$$\text{Area} = \frac{1}{9} \times \frac{1}{5} \times \frac{1}{2} = \frac{1}{90}$$

(3)

$$4\sqrt{3}\,\sin(3\theta+20) = 4\cos(3\theta+20)$$

$$\frac{\sin(3\theta+20)}{\cos(3\theta+20)} = \frac{4}{4\sqrt{3}}$$

$$\tan(3\theta+20) = \frac{1}{\sqrt{3}}$$

new interval $20 \leq 3\theta+20 \leq 560$

$$3\theta+20 = 30^\circ,\ 210^\circ,\ 390^\circ$$

$$\theta = \frac{10}{3},\ \frac{190}{3},\ \frac{270}{3}$$

to 1 decimal place

$$\theta = 3.3^\circ,\ 63.3^\circ,\ 123.3^\circ$$

(4) $\log_{11}(2x-1) = 1 - \log_{11}(x+4)$

get both logs on the same side

$$\log_{11}(2x-1) + \log_{11}(x+4) = 1$$

write as a single log (multiplication law)

$$\log_{11}(2x-1)(x+4) = 1$$

$$\log_{11}(2x^2+7x-4) = 1$$

write as an exponential

$$11^1 = 2x^2+7x-4$$

now solve

$$2x^2+7x-15=0$$

$$(2x-3)(x+5)=0$$

$$x = \frac{3}{2} \qquad x = -5$$

$x \neq 5$ as cannot log a negative

$\log_{11}(x+4)$

$\log_{11}(-5\ 4)$

$\log_{11}(-1)$

5

a)

$a + b$

$= 2pi + -5j + 6i - 3pj$

$= (2p+6)i + (-3p-5)j$

if parallel $\frac{-3p-5}{2p+6} = \frac{-5}{4}$

gradient of a+b

gradient of c

$-12p - 20 = -10p - 30$

$10 = 2p$

$p = 5$

b)

sub p=5 into a+b

$2(5)+6i + -3(5)-5j$

$16i - 20j$

6

a) $P = 100e^{0.4t}$

$t = 7$

$P = 100e^{0.4(7)} = 1644$

b) initial population when $t=0$ $P=100$

c) $100e^{0.4t} > 1000\,000$

$e^{0.4t} > 10000$

$\ln e^{0.4t} > \ln 10000$

$0.4t > \ln 10000$

$t > \frac{(\ln 10000)}{0.4}$

$t > 23.02585....$

$t = 24$ hours

⑦ $mx - y - 2 = 0$

$$y = mx - 2$$

$$x^2 + 6x + y^2 - 8y = 4$$

sub in $y = mx - 2$

$$x^2 + 6x + (mx-2)^2 - 8(mx-2) = 4$$

expand + simplify

$$x^2 + 6x + m^2x^2 - 12mx + 16 = 0$$

rewrite order x^2 x constant

$$x^2 + m^2x^2 + 6x - 12mx + 16 = 0$$

factorise out x^2 and x

$$(1+m^2)x^2 + (6x - 12m)x + 16 = 0$$

$$a = 1 + m^2$$
$$b = 6x - 12m$$
$$c = 16$$

$$b^2 - 4ac = 0$$

$$(6x - 12m)^2 - 4(1+m^2)(16) = 0$$

expand and simplify

$$20m^2 - 36m - 7 = 0$$

7 continued

$$20m^2 - 36m - 7 = 0$$

$a = 20$

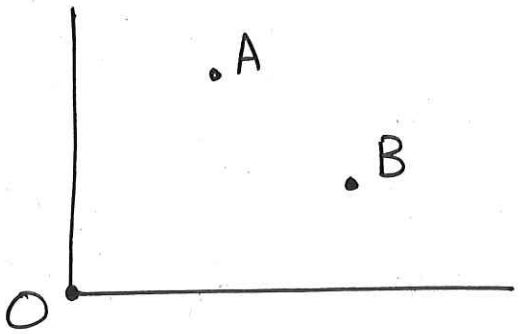
$b = -36$

$c = -7$

$$m = \frac{9 + 2\sqrt{29}}{10} \quad \text{or} \quad \frac{9 - 2\sqrt{29}}{10}$$

8

a)



$OA = 4i + 7j$

$OB = 10i + 9j$

$AB = AO + OB$

$OA \rightarrow AO$
change
signs

$AB = -4i - 7j + 10i + 9j$

$AB = 6i + (9-7)j$

(8) $AB = 6i + (q-7)j$

b) $|AB| = \sqrt{6^2 + (q-7)^2}$

$$2\sqrt{13} = \sqrt{36 + q^2 - 14q + 49}$$

$$(2\sqrt{13})^2 = q^2 - 14q + 85$$

$$4(13) = q^2 - 14q + 85$$

$$52 = q^2 - 14q + 85$$

$$0 = q^2 - 14q + 33$$

$$q = 11 \text{ or } 3$$

(9) $(2 + px)^9$

a)

$$= \binom{9}{0}(2)^9(px)^0 \qquad (1)(512)(1)$$

$$+ \binom{9}{1}(2)^8(px)^1 \qquad (9)(256)(px)$$

$$+ \binom{9}{2}(2)^7(px)^2 \qquad (36)(128)(p^2x^2)$$

$$+ \binom{9}{3}(2)^6(px)^3 \qquad (84)(64)(p^3x^3)$$

$$= 512 + 2304px + 4608p^2x^2$$

$$+ 5376p^3x^3$$

b)

$$p^3 = -84$$

$$p = -1/4$$

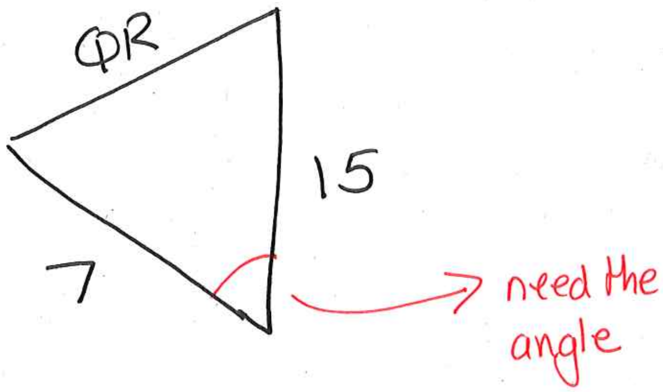
c)

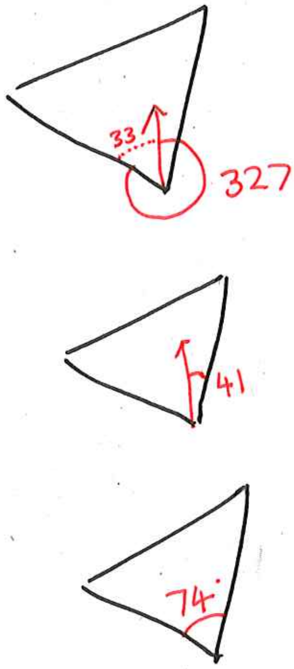
x^2 $\quad (-1/4)^2 \times 4608 = -576$

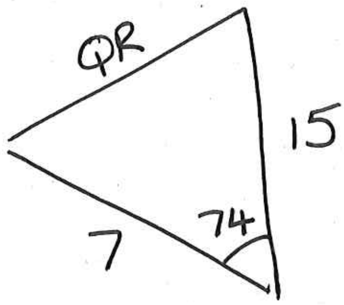
x $\quad (-1/4) \times 2304 = 288$

(10)

a)





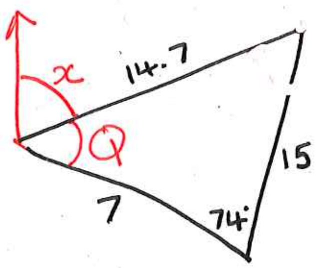


cosine rule

$QR^2 = 15^2 + 7^2 - 2 \times 15 \times 7 \cos 74$

$QR = 14.7$ km

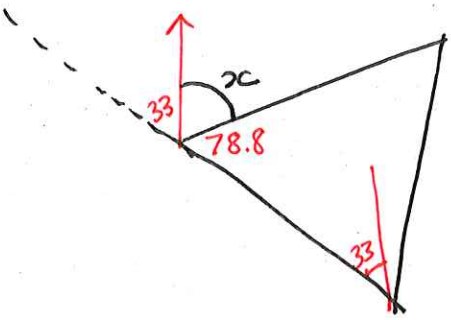
b)



sin rule

$$\frac{\sin Q}{15} = \frac{\sin 74}{14.7}$$

$Q = 78.8^\circ$



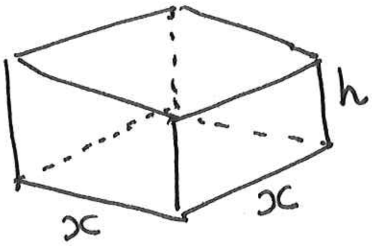
$x = 180 - 33 - 78.8$

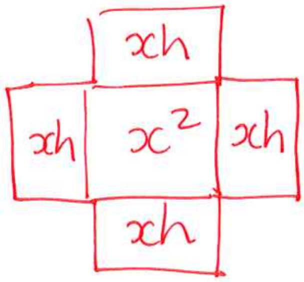
$x = 68.2$

Bearing $= 068^\circ$

11

a)





$$\text{Surface Area} = x^2 + 4xh$$

$$1600 = x^2 + 4xh$$

make h subject

$$\frac{1600 - x^2}{4x} = h$$

$$V = x^2 h$$

$$V = x^2\left(\frac{1600 - x^2}{4x}\right)$$

$$V = \frac{1600x^2 - x^4}{4x}$$

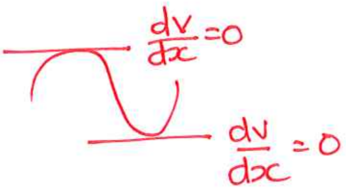
$$V = \frac{1600x^2}{4x} - \frac{x^4}{4x}$$

$$V = 400x - \frac{x^3}{4}$$

(11)

b) $$V = 400x - \frac{x^3}{4}$$

$$\frac{dV}{dx} = 400 - \frac{3x^2}{4}$$



$$0 = 400 - \frac{3x^2}{4}$$

$\times 4$

$$0 = 1600 - 3x^2$$

$$3x^2 - 1600 = 0$$

$$3x^2 = 1600$$

$$x^2 = \frac{1600}{3}$$

$$x = \sqrt{\frac{1600}{3}}$$

sub into $V = 400x - \frac{x^3}{4}$

$$V = 6160 \text{ (3sf)}$$

(11)

c) $$\frac{dV}{dx} = 400 - \frac{3x^2}{4}$$

$$\frac{d^2V}{dx^2} = -\frac{6x}{4}$$

$x > 0$

$\therefore \frac{d^2V}{dx^2} < 0$

$\therefore$ max value

(12)

a) $y = -x^3 + 2x^2 + 8x$

$y = 0$

$0 = -x^3 + 2x^2 + 8x$

$x^3 - 2x^2 - 8x = 0$

$x(x^2 - 2x - 8) = 0$

$x(x-4)(x+2) = 0$

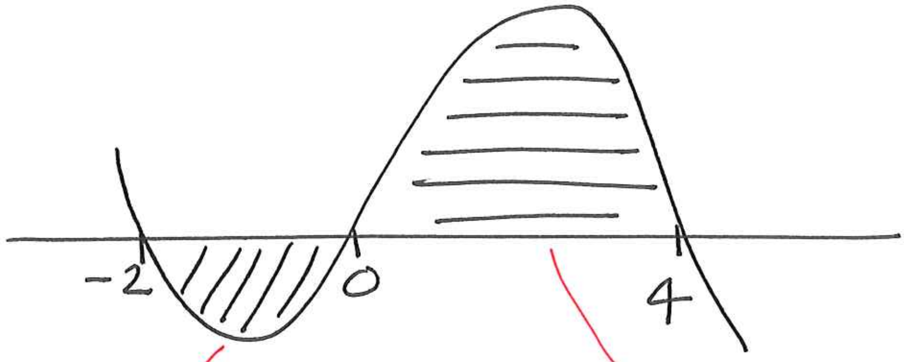
$x = 0, 4, -2$

A $(-2, 0)$ B $(4, 0)$

$8 + 8 - 16$

(12)

b)



$$\int_{-2}^{0} -x^3+2x^2+8x$$

$$\left[-\frac{x^4}{4}+\frac{2x^3}{3}+\frac{8x^2}{2}\right]_{-2}^{0}$$

$$[0]-[6\tfrac{2}{3}]$$

$$= -6\tfrac{2}{3}$$

Area $= 6\tfrac{2}{3}$

$$\int_{0}^{4} -x^3+2x^2+8x$$

$$\left[-\frac{x^4}{4}+\frac{2x^3}{3}+\frac{8x^2}{2}\right]_{0}^{4}$$

$$\left[\frac{128}{3}\right]-[0]$$

$$= \frac{128}{3}$$

Area $= \frac{128}{3}$

Total Area $= 6\tfrac{2}{3} + \frac{128}{3}$

$= 49\tfrac{1}{3}$

(13)

a) $x^2 - 10x - 20 = 0$

$(x-5)^2 - 25 - 20 = 0$

$(x-5)^2 = 45$

$x - 5 = \pm\sqrt{45}$

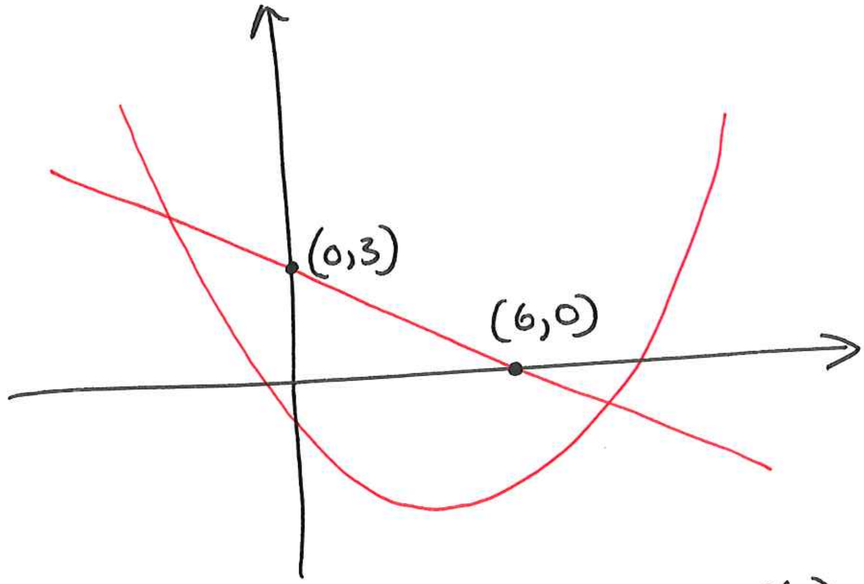
$x = 5 \pm \sqrt{45}$

$x = 5 \pm \sqrt{9}\sqrt{5}$

$x = 5 \pm 3\sqrt{5}$

11

b)



$P(x) = 3 - \frac{1}{2}x$

$x = 0 \quad y = 3$

and when

$y = 0 \quad x = 6$

c) $x^2 - 10x - 20 = 3 - \frac{1}{2}x$

$\times 2$

$2x^2 - 20x - 40 = 6 - x$

$2x^2 - 19x - 46 = 0$

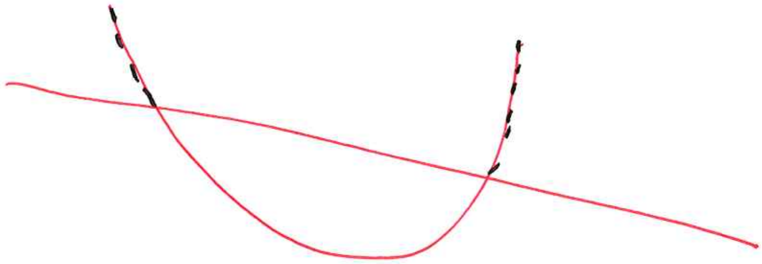
$x = -2, \frac{23}{2}$

$(-2, 4)$

$\left(\frac{23}{2}, \frac{11}{4}\right)$

(11)

d)



$x < -2$ $\qquad$ $x > \frac{23}{2}$

$x < -2$ or $x > \frac{23}{2}$