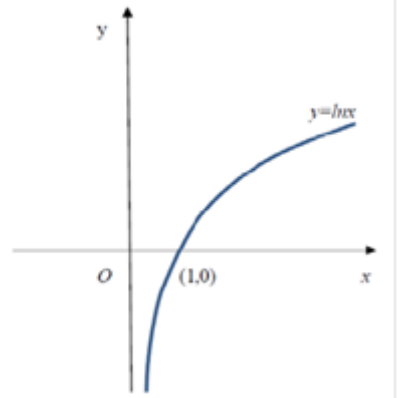
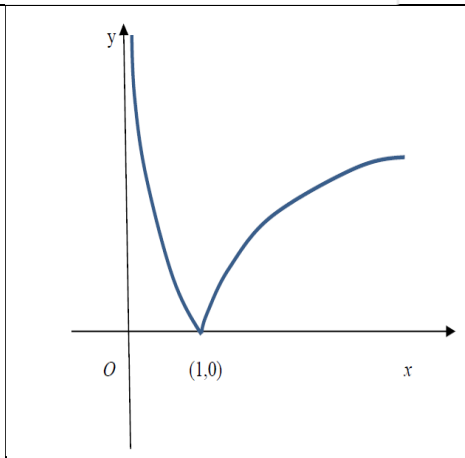
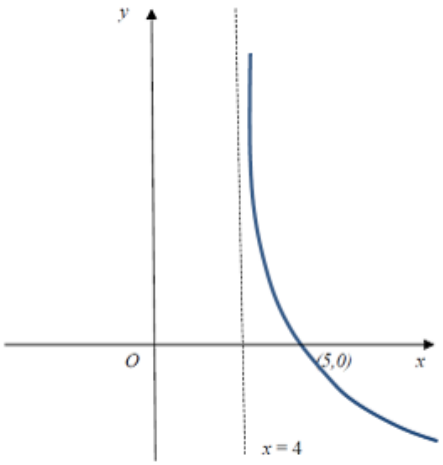
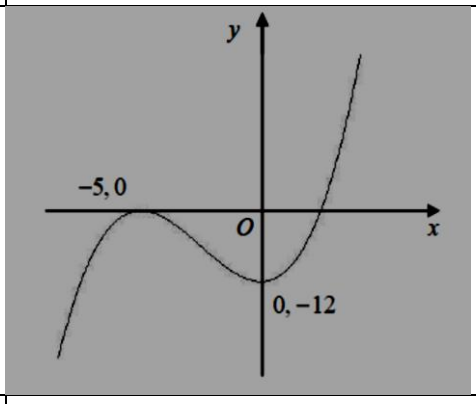
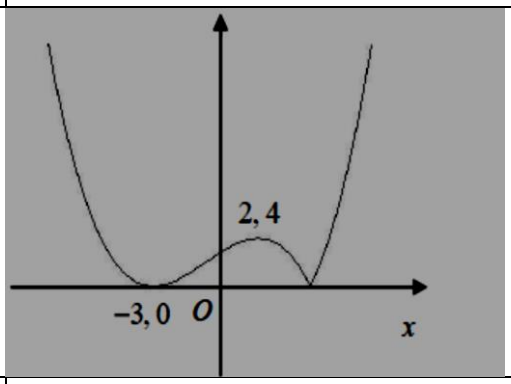
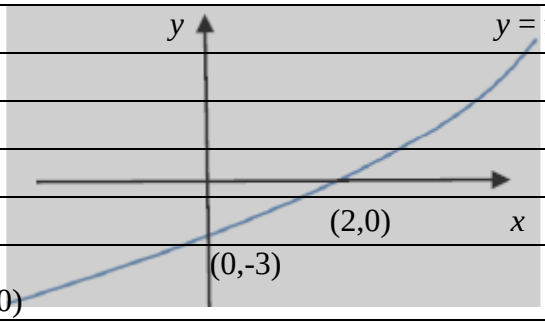
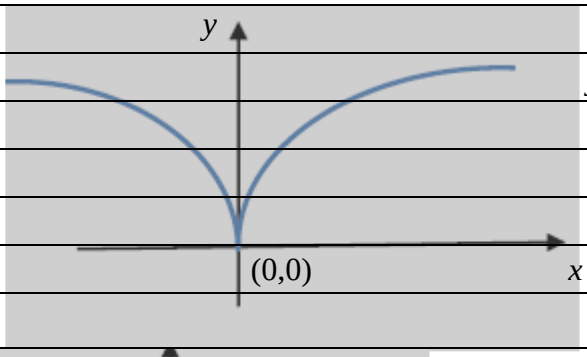
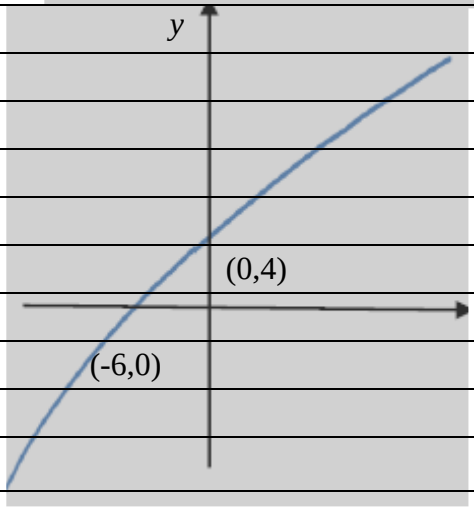
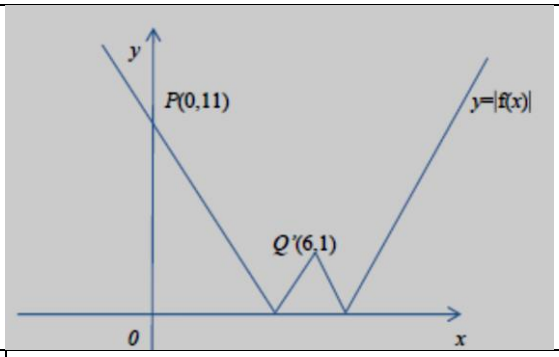
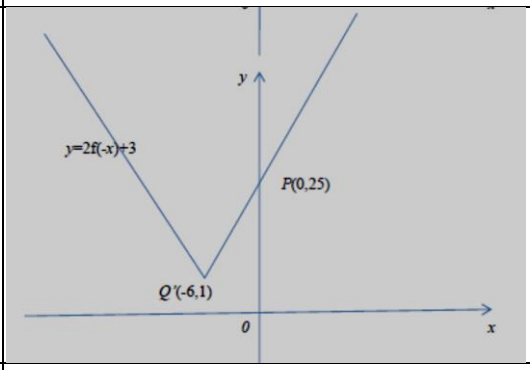
A level Mathematics Practice Paper – Algebra and functions – Mark scheme

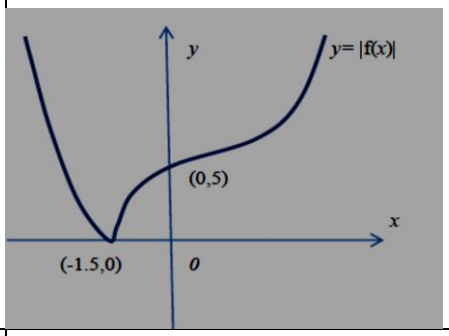
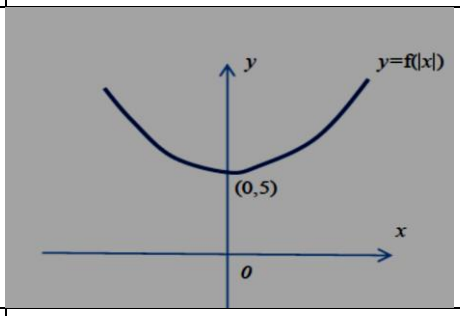
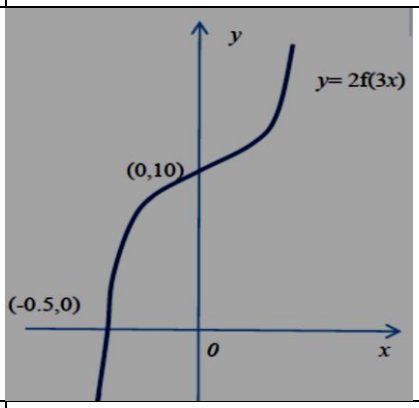
1(a)	$\text{fg}(x) = \dfrac{28}{x-2} - 1$	M1
	$\left(= \dfrac{30-x}{x-2}\right)$	
	Sets $\text{fg}(x) = x \Rightarrow \dfrac{28}{x-2} - 1 = x$	
	$\Rightarrow 28 = (x+1)(x-2)$	M1
	$\Rightarrow x^2 - x - 30 = 0$	
	$\Rightarrow (x-6)(x+5) = 0$	dM1
	$\Rightarrow x = 6$, $x = -5$	A1
		(4)
1(b)	$a = 6$	B1ft
		(1)
		(**5 marks)**

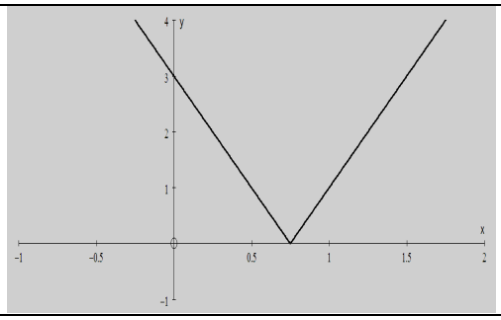
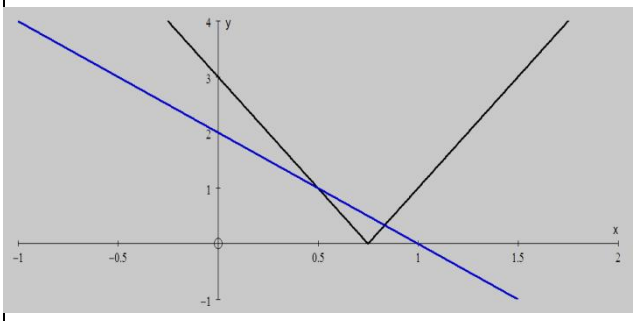
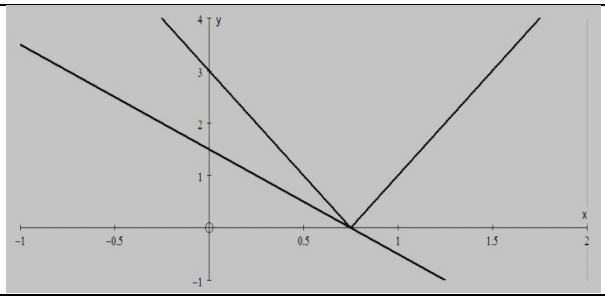
2		*ln* graph crossing *x* axis at (1,0) and asymptote at $x = 0$	B1
		Shape including cusp Touches or crosses the *x* axis at (1,0) Asymptote given as $x=0$	B1ft B1ft B1
		Shape Crosses at (5, 0) Asymptote given as $x = 4$	B1 B1ft B1
			(7 marks)

3(a)	$y \geqslant 3$	B1
		(1)
3(b)	$y = 3 + \sqrt{x+2} \Rightarrow y - 3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x \geqslant 3$	A1
		(3)
3(c)	$g(x) = x \Rightarrow 3 + \sqrt{x+2} = x$	
	$\Rightarrow x + 2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1 A1
	$\Rightarrow x = \frac{7 \pm \sqrt{21}}{2} \Rightarrow x = \frac{7 + \sqrt{21}}{2}$ only	M1 A1
		(4)
3(d)	$a = \frac{7 + \sqrt{21}}{2}$	B1 ft
		(1)
		(9 marks)
4(a)	 Shape *x* coordinates correct y coordinates correct	B1 B1 B1
		(3)
4(b)	 Shape Max at (2,4) Min at (-3,0)	B1 B1 B1
		(3)
		(6 marks)

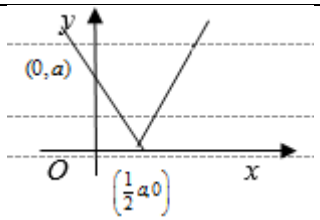
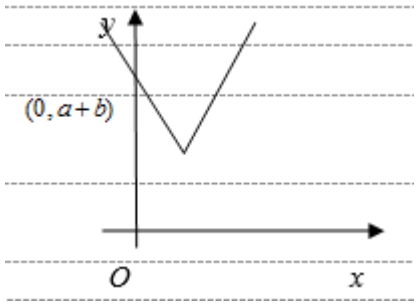
5(a)	ff(-3)= f(0),=2	M1 A1
		(2)
5(b)		
	Shape	B1
	(0,-3) and	B1
		(2)
5(c)		
	y=f(|x|)-2	
	Shape	B1
	(0,0)	B1
		(2)
5(d)		
	Shape	B1
	(-6,0) or (0,4)	B1
	(-6,0) and (0,4)	B1
		(3)
		(9 marks)

6(a)		'W' Shape (0, 11) and (6, 1)	B1 B1
			(2)
6(b)		'V' shape (-6,1) (0,25)	B1 B1 B1
			(3)
6(c)	One of $a = 2$ or $b = 6$		B1
	$a = 2$ and $b = 6$		B1
			(2)
			(7 marks)

7(a)		Shape including cusp (–1.5, 0) **and** (0, 5)	B1 B1
			(2)
7(b)		Shape (0,5)	B1 B1
			(2)
7(c)		Shape (0,10) (-0.5, 0)	B1 B1 B1
			(3)
			(7 marks)

8(a)	 V shaped graph Touches x axis at ¾ and cuts y axis at 3	B1 B1
		(2)
8(b)	 Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ or $x>\frac{5}{6}$ or $x<\frac{1}{2}$	M1 A1
	$x<\frac{1}{2}$ or $x>\frac{5}{6}$	dM1A1
		(4)
8(c)	 Draws graph Or solves $\lvert 4x-3\rvert=1\frac{1}{2}-2x$ to give one soln x = ¾	M1
	Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\mathbb{R},)\ x\neq\frac{3}{4}$, or $x<\frac{3}{4}$, $x>\frac{3}{4}$	A1
		(2)
		(8 marks)

9(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\frac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\frac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$=\frac{(x+1)}{(x-2)}$ cso	A1*
		(4)
9(b)	One end either $(y)>1,(y)\geqslant 1$ or $(y)<4,(y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
9(c)	Attempt to set	
	Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\frac{(x+1)}{(x-2)}=x$ $\frac{x+1}{x-2}=\frac{2x+1}{x-1}$ $\frac{2x+1}{x-1}=x$ $\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=...$	A1, dM1
	$a=\frac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4)
		(10 marks)

10(a)(i)		V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ **and** $(0,a)$ Correct shape, position and coordinates	B1 B1
10(a)(ii)		Their "V" shape translated up or $(0,a+b)$ Correct shape, position and $(0,a+b)$	B1ft B1
			(4)
10(b)	States or uses $a+b=8$		B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$		
	$2c-a+b=\frac{3}{2}c+8\Rightarrow kc=\text{f}(a,b)$		M1
	Combines $kc=\text{f}(a,b)$ with $a+b=8$ $\Rightarrow c=4a$		dM1 A1
			(4)
			(8 marks)