(1)

a)

$$\sin(x - 20) = \frac{1}{\sqrt{2}} \qquad 0 \leq x < 360$$

$$\sin y = \frac{1}{\sqrt{2}} \qquad -20 \leq y < 340$$

$$y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$$y = 180 - 45 = 135°$$

$$y = 45 + 360 = 405° \quad \text{out of range}$$

hence

$$x - 20 = 45 \qquad \therefore \quad x = 65°$$

$$x - 20 = 135 \qquad \therefore \quad x = 155°$$

$$x = 65°, \ 155°$$

(1)

b)

$$\cos(3x) = -\frac{1}{2} \qquad 0 \le x < 360$$

$$\cos(y) = -\frac{1}{2} \qquad 0 \le y < 1080$$

$$y = \cos^{-1}\left(-\frac{1}{2}\right) = 120°$$

$$y = 360 - 120 = 240°$$

$$y = 120 + 360 = 480°$$

$$y = 240 + 360 = 600°$$
$$y = 480 + 360 = 840$$
$$y = 600 + 360 = 960°$$

hence

$3x = 120$	$\therefore x = 40°$
$3x = 240$	$\therefore x = 80°$
$3x = 480$	$\therefore x = 160°$
$3x = 600$	$\therefore x = 200°$
$3x = 840$	$\therefore x = 280°$
$3x = 960$	$\therefore x = 320°$

(2)

a)

$$5 \sin \theta = 2 \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{2}{5}$$

$$\tan \theta = \frac{2}{5}$$

b)

$$5 \sin 2x = 2 \cos 2x$$

$$\tan 2x = \frac{2}{5} \qquad 0 \leq x < 360$$

$$\tan y = \frac{2}{5} \qquad 0 \leq y < 720$$

$$y = \tan^{-1}\left(\frac{2}{5}\right) = 21.8$$

$$y = 180 + 21.8 = 201.8$$

$$y = 180 + 201.8 = 381.8$$

$$y = 180 + 381.8 = 561.8$$

$$2x = 21.8 \qquad \therefore x = 10.9°$$

$$2x = 201.8 \qquad \therefore x = 100.9°$$

$$2x = 381.8 \qquad \therefore x = 190.9°$$

$$2x = 561.8 \qquad \therefore x = 280.9°$$

③

a)

$$5 \sin x = 1 + 2 \cos^2 x$$

$$5 \sin x = 1 + 2(1 - \sin^2 x)$$

$$5 \sin x = 1 + 2 - 2 \sin^2 x$$

$$5 \sin x = 3 - 2 \sin^2 x$$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

let $\sin x = y$

$$2y^2 + 5y - 3 = 0$$

$$(2y - 1)(y + 3) = 0$$

$$y = \frac{1}{2} \qquad y = -3$$

$$\sin x = \frac{1}{2} \qquad\qquad \sin x = -3$$

$$\qquad\qquad\qquad\qquad\qquad \text{No solutions}$$

$$x = 30$$

$$x = 180 - 30 = 150$$

$$x = 30°, 150°$$

(4)

$$2\cos^2 x + 1 = 5\sin x$$

$$2(1 - \sin^2 x) + 1 = 5\sin x$$

$$2 - 2\sin^2 x + 1 = 5\sin x$$

$$0 = 2\sin^2 x + 5\sin x - 3$$

let $y = \sin x$

$$0 = 2y^2 + 5y - 3$$

$$0 = (2y - 1)(y + 3)$$

$$y = \tfrac{1}{2} \quad y = -3$$

$$\sin x = \frac{1}{2} \qquad \sin x = -3$$

No solutions

$$x = 30°$$

$$x = 180 - 30° = 150°$$

$$x = 30°, \ 150°$$

⑤

a)

$$4\sin^2 x + 9\cos x - 6 = 0$$

$$4(1 - \cos^2 x) + 9\cos x - 6 = 0$$

$$4 - 4\cos^2 x + 9\cos x - 6 = 0$$

$$0 = 4\cos^2 x - 9\cos x + 2$$

b)

let $\cos x = y$

$$0 = 4y^2 - 9y + 2 \qquad 0 \leq x < 720$$

$$0 = (4y - 1)(y - 2)$$

$$y = \frac{1}{4} \qquad y = 2$$

$$\cos x = \frac{1}{4} \qquad\qquad \cos x = 2$$

No solutions

$$x = 75.5°$$

$$x = 360 - 75.5 = 284.5°$$

$$x = 75.5 + 360 = 435.5°$$

$$x = 284.5 + 360 = 644.5°$$

$$x = 75.5°, \ 284.5°, \ 435.5°, \ 644.5°$$

⑥

i) $(1 + \tan\theta)(5\sin\theta - 2) = 0$ $-180 \leq \theta < 180$

$1 + \tan\theta = 0$ $5\sin\theta - 2 = 0$

$\tan\theta = -1$ $\sin\theta = \dfrac{2}{5}$

$\theta = -45°$ $\theta = 23.6°$

$\theta = -45 + 180 = 135°$ $\theta = 180 - 23.6 = 156.4°$

$\theta = -45°, 23.6°, 135°, 156.4°$

ii)

$4\sin x = 3\tan x$

$4\sin x = \dfrac{3\sin x}{\cos x}$

$4\sin x \cos x = 3\sin x$

$4\sin x \cos x - 3\sin x = 0$

$\sin x(4\cos x - 3) = 0$ $0 \leq x < 360$

$\sin x = 0$ $4\cos x - 3 = 0$

                    $\cos x = \dfrac{3}{4}$

$x = 0°$

$x = 180 - 0 = 180°$ $x = 41.4°$

                       $x = 360 - 41.4 = 318.6$

$x = 0°, 41.4°, 180°, 318.6°$